Sub-λ/10 spot size in semiconductor solid immersion lens microscopy

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A R T I C L E   I N F O
Article history:
Received 26 September 2013
Received in revised form 23 October 2013
Accepted 6 November 2013
Available online 19 November 2013

Keywords:
Optical microscopy
High-resolution imaging
Solid immersion lens
Apodization
Particle swarm optimization

A B S T R A C T

The angular spectrum of radially polarized Laguerre–Gaussian beams was tailored by an annular aperture in a semiconductor solid immersion lens microscope. The radii of two concentric rings in the amplitude aperture were optimized by a multi-objective particle swarm optimization algorithm. A GaAs solid immersion lens with NA=3.4 was used in the numerical calculation and a spot size of 98 nm (<λ/10) at a wavelength of 1064 nm illumination was obtained.

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1. Introduction

As the feature sizes in integrated circuits shrink, improving the optical resolution to localize features becomes more crucial in optical semiconductor fault analysis [1]. Remarkable advances in super-resolution fluorescence microscopy allow imaging with a spatial resolution below 20 nm [2,3], but they are limited to light-emitting objects and are not suitable for direct imaging, and especially subsurface imaging.

High numerical aperture solid immersion lenses (SILs) are well known to enhance the resolution and collection efficiency in backside infrared subsurface microscopy [4,5]. Using a semiconductor SIL made of Si or GaAs removes the reflection at the lens–substrate interface and it improves the system NA up to 3.5. Here we studied the optimization of the solid immersion lens apodization illuminated by radially polarized Laguerre–Gaussian beams. Laguerre–Gaussian beams are high-order laser beams with ring-shaped intensity profile and have applications in optical trapping [6] and laser micromachining [7]. These higher-order laser beams have the unique property that there is a π phase shift between the electric field of the subsequent rings [8]. The destructive interference between these field components provides tighter spot sizes under high NA focusing. Incorporation of radial polarization and ring shape beam profile can improve the focusing even more [9,10].

To minimize the spot size and control the intensity profile at the focal plane, the geometry of the amplitude annular aperture was optimized by a particle swarm optimization algorithm [11]. Particle swarm optimization (PSO) is a nature-inspired optimization algorithm in which each particle represents a potential solution in a multi-dimensional space. A candidate solution in PSO gradually moves toward a global best point and each particle adjust its position according to its own experience and those neighboring particles. We obtained a spot size of 98 nm with illumination at 1064 nm using a multi-objective PSO algorithm. Fig. 1 shows the schematics of the beam apodization approach in a SIL microscope.

2. Higher-order laser beams

The electric field of a Laguerre–Gaussian beam can be expressed as [12]:

\[ E_{L_{n}^{m}}(\rho, \phi, z) = \left( \frac{2n!}{\prod (n + m)!} \frac{1}{\omega(z)} \right)^{m} \exp \left( -\frac{\rho^{2}}{\omega^{2}(z)} \right) I_{n}^{m} \left( \frac{2\rho^{2}}{\omega^{2}(z)} \right) \exp \left( -\frac{i(kz^{2})}{2R(z)} \right) \times \exp \left( -i(2n+m+1) \arctan \left( \frac{z}{R(z)} \right) \right) e^{-im\phi} , \]

where \( \omega(z) \) and \( R(z) \) are the beam waist and radius of curvature for the Gaussian component, respectively, \( L_{n}^{m}(x) \) is the generalized Laguerre polynomial, and \( z_{0} = \frac{\pi \omega^{2}}{4} / \lambda \) is the Rayleigh range of the Gaussian beam. The normalization coefficient was set such that the power of the beam is normalized to one. It can be shown that the intensity distribution of \( E_{m}^{LC} \) beams consists of \( n+1 \) concentric rings and for \( n \geq 1 \) the electric field of the subsequent rings have a π phase shift with respect to each other [8]. Here we used the lowest order of Laguerre–Gaussian beams, \( E_{11}^{LC} \), consisting of at least

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two rings in the radial intensity profile. Similar calculations can be carried out for higher order modes. Working with lower order Laguerre–Gaussian beams simplifies the mask pattern and makes the creation of the incident beam easier to implement experimentally.

Fig. 2a shows the field profile of \( E_{11}^{LG} \) beam and Fig. 2b shows the longitudinal and total intensity profile of this beam at the focal plane of a GaAs SIL with NA = 3.4 and illuminated at 1064 nm. The full width at half maximum (FWHM) of the total intensity is 140 nm.

The beam waist for a Gaussian beam is defined such that the electric field drops to 1/e of the maximum amplitude at the beam waist. For a Laguerre–Gaussian beam, the spot size radius was defined as the square root of \[\alpha^2(z) = \frac{2}{\omega_0} \int_0^{\infty} \int_0^\pi r^2 I_{mm}(\rho, \phi, z) \rho \, d\rho \, d\phi. \] (2)

It can be shown that the beam waist of a Laguerre–Gaussian beam simplifies to
\[\alpha_{\text{mm}}^2(0) = \frac{1}{2\pi m + 1}, \]
with \( n = m = 0 \) the Laguerre–Gaussian beam becomes a Gaussian beam and the spot size becomes \( \alpha(0) \). By introducing \( \beta_0 = \alpha_{\text{mm}}(0)/\alpha_0 \), we have \( \beta_0 = 2 \) for \( E_{11}^{LG} \) beam. The coefficient \( \beta_0 \) is the ratio of the SIL entrance pupil to the incident beam waist. Therefore, with \( f \) being the radius of the SIL and \( \theta_{\text{max}} = \sin^{-1}(\text{NA}/n_{\text{GaAs}}) \), we have \( \beta_0 = f \sin \theta_{\text{max}} \).

As Fig. 3 shows, the spot size of \( E_{11}^{LG} \) beam is very sensitive to the beam size filling the SIL aperture. Fig. 4 shows the intensity distribution of \( E_{11}^{LG} \) beam at the focal plane of our SIL.

The central lobe has 73 nm spot size, but the side lobes have much higher intensity than the central peak. To improve the relative intensity of the central peak with respect to the side lobes and keeping the spot size minimum, we have used a multi-objective particle swarm optimization (MOPSO) algorithm.

3. Multi-objective particle swarm optimization

In PSO algorithms a number of particles (candidate solutions) move in the search space and the goal is to find a solution, which minimizes an objective function. The movement of each particle for the next iteration is determined by its movement through the search space based on its own best and the global best of all other particles. For an \( n \)-dimensional design space and a swarm of \( m \) particles, the velocity \( v_i = (v_{i1}, v_{i2}, \ldots, v_{in}) \) and position \( x_i = (x_{i1}, x_{i2}, \ldots, x_{in}) \) of \( i \)-th particle at \( k \)-th iteration is updated according to
\[v_i^{k+1} = W v_i^k + c_1 \times r_1 \times (\text{best}_i^k - x_i^k) + c_2 \times r_2 \times (\text{Gbest}_k^k - x_i^k), \]
\[x_i^{k+1} = x_i^k + v_i^{k+1}, \quad i = 1, \ldots, m, \] (5)

![Fig. 1](image1.png) **Fig. 1.** Apodization of a solid immersion lens illuminated by the radially polarized Laguerre–Gaussian beams.

![Fig. 2](image2.png) **Fig. 2.** (a) The field profile of \( E_{11}^{LG} \) beam. (b) The intensity profile of the longitudinal and total intensity of this beam at the focal plane of SIL.

![Fig. 3](image3.png) **Fig. 3.** The variation of \( E_{11}^{LG} \) beam spot size versus \( \beta_0 \). At \( \beta_0 = 1.34 \) the spot size is at its minimum.
where $p_{best}$ is the best position experienced by the particle in the design space and $G_{best}$ is the best position visited by neighboring particles. $W$ is the inertial weight and $c_1$ and $c_2$ are the positive constants, which represent the acceleration to shift the particle velocity toward $p_{best}$ and $G_{best}$ respectively. In addition, $r_1$ and $r_2$ are the random numbers uniformly distributed in the range of $[0, 1]$. The PSO algorithm starts with random numbers for $x$, $v$, $p_{best}$ and $G_{best}$ and it continues until a certain iteration number is reached or when certain minimization criteria are fulfilled.

Our amplitude mask design consists of several objectives and constraints. We need smallest spot size for the central peak, the ratio for the intensity of the central peak with respect to side lobe intensities has to be maximized, and the central peak should be well separated from side lobe field distribution. In a MOPSO, often multiple objectives are directly conflicting, which makes identification of the local and global best solutions difficult. The concept of non-dominance was introduced to resolve this problem [15]. Non-dominance solutions are the solutions in which the objective functions have their lower values with respect to the other solutions. Therefore instead of having just one individual solution as the global best, a set of all the non-dominated solutions is maintained in the form of an archive. Since members of the archive are globally better than other members of the swarm, we only need to find a way to choose a global guide for each particle in the swarm from the archive [15]. The non-dominance archive in our calculation was updated at the end of each iteration step and the $G_{best}$ solution was chosen from the sorted solution in the archive.

We used an adaptive MOPSO, in which the $W$, $c_1$ and $c_2$ coefficients were adjusted at each iteration step such that the values of these coefficients starts from 0.9, 2.0, and 2.0 respectively and reach to 0.5, 1.5, and 1.5 at the end of the iteration loop. The adaptive approach increases the exploration speed of the solutions in the search space and helps the $G_{best}$ solution converge faster.

4. Annular apodization

An amplitude mask was introduced between the incident beam and the SIL Fig. 1. The objective functions in our MOPSO algorithm were minimization of the spot size and maximizing the relative intensity of central spot with respect to the side lobe intensities. The vector diffraction theory of an aplanatic lens was used to calculate these functions [16]. A mask with two transparent regions (rings) was chosen and the angles between the optical axis and the perimeter of these two concentric rings were used as optimization parameters. An adaptive MOPSO code with 40 particles was used in the calculation. Fig. 5a shows the profiles of total and longitudinal intensities. The $\beta_0$ coefficient was set to 1.34 for these calculations. As the figure shows the intensity at central peak is dominated by the longitudinal component, but the radial component is more pronounced at the side lobes. The total and longitudinal intensities were normalized to the maximum total intensity when there was no mask in the optical path.

Full width at half maximum of the central spot was 98 nm ($\lambda/10.85$ at $\lambda = 1064$ nm). Although the spot size slightly increased from 73 to 98 nm, the central peak is now significantly more dominant in the intensity distribution. Fig. 5b shows the total intensity distribution at the focal plane. As this figure shows, the side lobes were well separated from the central spot, which helps to aperture them out at the image plane. Our optimization method maximizes the separation between the central lobe and sidelobes, such that the first minima around the central lobe approach to zero, Fig. 5a. The scattered light from the separated sidelobes can be effectively suppressed by using a suitable confocal aperture in the collection arm of the SIL microscope [17]. The angles defining the first and second rings in our optical mask were $\theta_1 = 0.440_{\max}$, $\theta_2 = 0.630_{\max}$ and $\theta_3 = 0.850_{\max}$. $\theta_4 = \theta_{\max}$ respectively, where $\theta_{\max} = \sin^{-1}(NA/n_{GaAs})$ (with $n_{GaAs} = 3.48$ at 1064 nm) is the half angle of the entrance pupil of the solid immersion lens.

![Fig. 4. (a) Intensity profile of $E_{1_{LG}}$ beam with $\beta_0 = 1.34$ at the focal plane of SIL. (b) The intensity distribution.](image1)

![Fig. 5. (a) Longitudinal and total intensities at the focal plane of SIL after apodization. (b) Intensity distribution at the focal plane.](image2)
5. Conclusion

The optimization of the apodization using the multiple-objective particle swarm optimization was carried out in a solid immersion lens microscopy. The calculation was based on a solid immersion lens made of GaAs with $\text{NA} = 3.4$ and illumination wavelength was at 1064 nm. The incident beam was a radially polarized Laguerre–Gaussian beam with $m = n = 1$ order. The calculation showed that although by using the beam parameter of $\beta_0 = 1.34$ the spot size can be reached to 73 nm, but to maximize the central spot intensity with respect to side lobes an amplitude mask should be used. The geometry of the mask was optimized by an adaptive multiple-objective particle swarm optimization and the spot size of 98 nm was obtained.

Acknowledgments

The work presented in this paper was supported by the Intelligence Advanced Research Projects Activity (IARPA) via Air Force Research Laboratory (AFRL) Contract no. FA8650-11-C-7102.

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